DI Physics Hons.

Paper-I

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Relations between the Elastic Constants

The elastic constants are dependent to each other, since any change in the size and shape of a body may be obtained by first changing the size of the body only and then by changing the shape only. Thus, the expressions can be derived, showing the inter-relations between them.

(i) Relations between Y, K and σ:

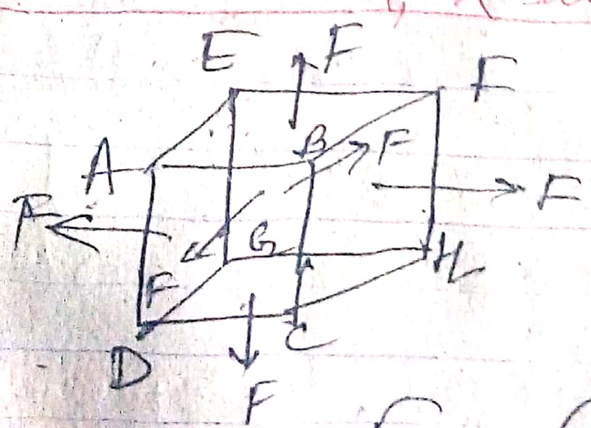

fig. (1)

Let ABCDEFGH represent a cube of unit side. Let us consider a force F, which acts normally and uniformly on each of its six faces in the outward direction.

If α is the increase per unit length per unit tension along the direction of the force, then the elongation produced in each of the edges, namely AB, BF and BC, will be $F.\alpha$. If β is the contraction produced per unit length per unit tension perpendicular to the edges, then contraction produced perpendicular to each of the edge, namely AB, BF and BC, will be $F.\beta$. Thus, the sides of the cube becomes

$$AB = 1 + F\alpha - F\beta - F\beta = 1 + F(\alpha - 2\beta)$$

$$BF = 1 + F\alpha - F\beta - F\beta = 1 + F(\alpha - 2\beta)$$

$$BC = 1 + F\alpha - F\beta - F\beta = 1 + F(\alpha - 2\beta)$$

Hence, the final volume of the cube is

$$AB \times BF \times BC = [1 + F(\alpha - 2\beta)]^3$$

$$= 1 + 3F(\alpha - 2\beta)$$

[according to Bionomial expansion]

The terms containing higher powers of α and β heave been neglected, Since α and β are very small quantities.

$\therefore$ change in volume = final volume − initial volume

$$= 1 + 3F(\alpha - 2\beta) - 1 = 3F(\alpha - 2\beta).$$

$$\text{Volumetric Strain} = \frac{3F(\alpha-2\beta)}{1} = 3F(\alpha - 2\beta)$$

$$\therefore \text{bulk modulus } K = \frac{\text{normal stress}}{\text{Volumetric Strain}} = \frac{F}{3F(\alpha-2\beta)}$$

$$\text{or, } K = \frac{1}{3(\alpha - 2\beta)} \Rightarrow \text{or, } K = \frac{1/\alpha}{3(1-2\beta/\alpha)}.$$

But α is the increase per unit length for unit tension, hence clearly stress = 1 (as the area of any edge of the cube is unity) and

$$\text{linear strain} = \frac{\alpha}{1} = \alpha, \quad \therefore Y = 1/\alpha$$

Also,

$$\beta/\alpha = \sigma = \text{Poisson's ratio.}$$

Therefore;

$$K = \frac{Y}{3(1-2\sigma)} \quad \text{--- (i)}$$